Seat No. _____

# HB-003-2016003

## B. Sc. (Sem. VI) (CBCS) (W.E.F. 2019) Examination

**April - 2023**

## Mathematics : Paper - 10(A)

***(Optimization & Numerical Analysis - II)***

**Faculty Code : 003**
**Subject Code : 2016003**

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

**Instructions :** (1) Answer all questions.
(2) Figures to the right indicate marks.

**1** (a) Answer the following questions in short : **4**

(1) Which variable is introduce to convert the constraint $2\,2x+3y\le 5$ into equality?

(2) Define surplus variables with respect to LPP.

(3) Define convex set.

(4) Define convex linear combination.

(b) Attempt any one out of two : **2**

(1) Write the canonical form of LPP.

(2) Write the general mathematical form of LPP.

(c) Attempt any one out of two : **3**

(1) Explain graphical method for solving LPP.

(2) Obtain dual of the following LPP :

Min. $Z = 3x_1 + x_2$.

Sub. to $2x_1 + 3x_2 \ge 2$

$x_1 + x_2 \ge 1$

and $x_1, x_2 \ge 0$

(d) Attempt any one out of two : **5**

(1) Explain Two-Phase method for solving LPP.

(2) Explain simplex method for solving LPP.

**2** (a) Answer the following questions in short : **4**

(1) Who invented Hungerian method for solving Assignment problem?

(2) Write full form of LCM.

(3) Define non degenerate basic feasible solution with respect to Transportation problem.

(4) Write first step of NWCM to find initial solution of TP.

(b) Attempt any one out of two : **2**

(1) Write mathematical form of a Assignment problem.

(2) Write mathematical form of a transportation problem.

(c) Attempt any one out of two : **3**

(1) Explain LCM method for finding initial solution of Transportation problem.

(2) Find initial solution of following TP by NWCM method:

| | W | X | Y | Z | Supply |
|---|---|---|---|---|---|
| A | 19 | 30 | 50 | 10 | 7 |
| B | 70 | 30 | 40 | 60 | 9 |
| C | 40 | 8 | 70 | 20 | 18 |
| Demand | 5 | 8 | 7 | 14 | 34 |

(d) Attempt any one out of two : **5**

(1) Explain Hungarian method for solving Assignment problem.

(2) Find optimum solution of following Transportation problem:

| | W1 | W2 | W3 | Supply |
|---|---|---|---|---|
| F1 | 2 | 7 | 4 | 5 |
| F2 | 3 | 3 | 1 | 8 |
| F3 | 5 | 4 | 7 | 7 |
| F4 | 1 | 6 | 2 | 14 |
| Demand | 7 | 9 | 18 | 34 |

**3** (a) Answer the following questions in short : **4**

(1) Which formula is used for inverse interpolation?

(2) If $f(1, 3) = 5$, then find $f(3, 1)$.

(3) Write Laplace Everett's formula.

(4) Write Bessel's formula.

(b) Attempt any one out of two: **2**

(1) Define divided difference.

(2) If $f(x) = x^3$, then find $f(1, 3, 5)$.

(c) Attempt any one out of two : **3**

(1) Derive Bessel's formula.

(2) Apply lagrange's formula to find $f(2)$ given that $f(0) = -12$, $f(1) = 0$, $f(3) = 6$, $f(4) = 12$

(d) Attempt any one out of two: **5**

(1) Derive Gauss backward interpolation formula.

(2) Apply Laplace Everett's formula to find $y_{34}$ given that $y_{20} = 11.4699$, $y_{25} = 12.7834$, $y_{30} = 13.7648$, $y_{35} = 14.4982$, $y_{40} = 15.0463$.

**4** (a) Answer the following questions in short : **4**

(1) Write general quadrature formula.

(2) Write Trapezoidal Rule.

(3) Write Simpson's 3/8 Rule.

(4) Which formula is derived by taking $n = 2$ in general quadrature formula?

(b) Attempt any one out of two: **2**

(1) If $E = e^{hD}$, then show that

$$D^2 = \frac{1}{h^2}\left[\Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 - \frac{5}{6}\Delta^5 + .....\right]$$

(2) Evaluate $\int_2^6 \frac{dx}{x}$ by Simpson's 1/3 rule.

(c) Attempt any one out of two: **3**

(1) Find the value of $f'(90)$ using sterling's formula to the following data:

| $x$ | 60 | 75 | 90 | 105 | 120 |
|---|---|---|---|---|---|
| $f(x)$ | 28.2 | 38.2 | 43.2 | 40.9 | 37.7 |

(2) Derive Newton-Cote's quadrature formula.

(d) Attempt any one out of two: 5

(1) Derive Simpson's 3/8 rule.

(2) Evaluate $\int_0^{10} \frac{dx}{1+x^2}$ by (1) Trapezoidal rule (2) Simpson's 1/3 rule (3) Simpson's 3/8 rule.

**5** (a) Answer the following questions in short : **4**

(1) Write formula for Range's method.

(2) Write formula for Picard's method.

(3) Write formula for Euler's method.

(4) Write formula for Taylor's method.

(b) Attempt any one out of two: **2**

(1) For differential equation $\frac{dy}{dx} = 3x + y^2$, $y(1) = 1.2$, find $k_1, k_2$ by Range's method, $h = 0.1$.

(2) Find value of $y$ at $x = 0.1$ for first approximation by Picard's method of $\frac{dy}{dx} = 1 + xy, y(0) = 1$.

(c) Attempt any one out of two: **3**

(1) Explain Euler's modified method for solving

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0$$

(2) Explain Taylor's method for solving

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0$$

(d) Attempt any one out of two: **5**

(1) Explain Picard's method for solving

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0$$

(2) Solve $\frac{dy}{dx} = x^2 - y, y(0) = 1$. Find $y(0.1)$ by Runge kutta's method.